

# Electromagnetic form factors of the $\Delta$ in a S-wave approach

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Without any further adjusting of parameters, a relativistic constituent quark model, successful in the description of the data for the nucleon elastic form factors and of the dominant contribution for the nucleon to  $\Delta$  electromagnetic transition, is used here to predict the dominant electromagnetic form factors of the  $\Delta$  baryon. The model is based on a simple  $\Delta$  wave function corresponding to a quark-diquark system in an S-state. The results for E0 and M1 are consistent both with experimental results and lattice calculations. The remaining form factors E2 and M3 vanish, given the symmetric structure taken for the  $\Delta$ .

## I. INTRODUCTION

The internal quark structure of the nucleon can explain the experimental results for its electromagnetic form factors. For instance, in Ref. [1, 2], we presented a successful description of the most recent results from Jlab for the proton elastic form factors [3]. Also, in the last few years there has been an increasing convergence between the lattice QCD extrapolations and the low momentum ( $Q^2 < 2 \text{ GeV}^2$ ) form factor data [4, 5]. In principle the  $\Delta$  structure could also be inferred from the  $\gamma N \rightarrow \Delta$  electromagnetic transition [4, 5] (throughout this paper we will use  $N\Delta$  for short to name this reaction). However, models based on quark degrees of freedom alone fail in the description of the dominant multipole (magnetic dipole  $G_M^*$ ) of the  $N\Delta$  transition [6, 7]. Only phenomenological models including also explicit pion degrees of freedom, by adding coupled baryon-meson channels to the constituent quark structure, can describe this transition [8, 9]. Nevertheless, there are uncertainties because there is no unique way of separating the pion cloud from the short distance quark core effects. [10].

As for lattice calculations, at present they are not yet successful in describing the  $N\Delta$  electromagnetic form factors. Linear extrapolations to the physical limit of quenched calculations underestimate the data for low  $Q^2$  and overestimate the experimental data for larger  $Q^2$  [11]. The available unquenched lattice QCD results are not sufficient for an extrapolation to the physical region [12]. On the other hand, extrapolations of the lattice results using Chiral Perturbation Theory ( $\chi$ PT) were until now restricted to very low  $Q^2$  [13, 14]. Additionally, no  $Q^2$  dependence of the form factors was studied using  $\chi$ PT. It is therefore important to understand the structure of the  $\Delta$ , in particular the quark distribution inside the  $\Delta$  system, and to parametrize its electromagnetic form factors as functions of  $Q^2$ .

However, due to the short  $\Delta$  mean life time, there is almost no experimental information about the  $\Delta$  form factors, to the exception of the electrical charge. The magnetic dipole form factor was measured at  $Q^2 = 0$  only, giving the magnetic moment  $\mu_\Delta$  of the  $\Delta^{++}$ , since  $\mu_\Delta = G_{M1}(0) \frac{e}{2M_\Delta}$  (where  $M_\Delta$  is the  $\Delta$  mass and  $e$  the electron charge). Our knowledge of the  $\Delta^{++}$  dipole magnetic moment is essentially based on the experiments of Refs. [15, 16]. The results of the analysis are not all totally consistent [17] nevertheless. The Particle Data Group [18] gives a very broad interval for the  $\Delta^{++}$  magnetic moment,  $[3.7, 7.5]\mu_N$ , where  $\mu_N$  is the nucleon magneton. The recent analysis using a Dynamical Model [19] quotes  $\mu_\Delta = (6.14 \pm 0.51)\mu_N$ , but the theoretical error is not included. As for the  $\Delta^+$ , the only available measurement is from Mainz [20],  $\mu_\Delta = (2.7_{-3.6}^{+3.5})\mu_N$ , which still has a large error bar, mostly due to the theoretical uncertainties. However, new experiments are planned, and methods based on a Chiral Effective-field theory [21] and Dynamical Models [22] were proposed to extract the magnetic moment of  $\Delta^+$  from the reaction  $\gamma p \rightarrow \pi^0 p \gamma'$ . There is no data related to the  $\Delta^0$  and  $\Delta^-$  (see Refs. [19, 23] for a review).

In order to understand the present disagreement between the lattice calculations and the experimental results for the electromagnetic  $N\Delta$  transition, theoretical constituent quark models are promising tools. Given the absence of experimental data for the  $\Delta$ , they are needed to guide lattice extrapolations. Inversely it is crucial to probe models away from the physical limit by lattice data.

Models allow the calculation of the quark distribution inside the  $\Delta$  and generate predictions for the form factors. For the electrical quadrupole (E2) there are calculations by Buchmann based on several models [24]. For the magnetic moments there is also a variety of predictions based on several frameworks, such as the SU(6) quark model [22], relativistic quark models [25], quark models with two-body exchange currents [26] and sea quark contributions [27], QCD sum rules [28, 29], Chiral and Soliton models [30, 31, 32, 33], field theoretical formulations with hadron degrees

of freedom [34] and quark degrees of freedom [35], and extrapolations of lattice QCD [13, 36, 37, 38, 39]. We present a summary of these results in one of the following sections (Table I). Almost all works are focused on the  $\Delta$  magnetic moments. The exceptions to this rule are the works of Alexandrou *et. al.*, where the dependence of  $\Delta$  form factors on  $Q^2$  was studied systematically for the first time [39, 40], and the recent work of the Adelaide group [41].

Our work uses a covariant model [6] based on valence quark degrees of freedom to calculate the  $\gamma\Delta \rightarrow \Delta$  form factors and compares them to state-of-the-art results. We have restricted the nucleon and the  $\Delta$  wave functions to their S-wave components. With such a symmetric distribution only the charge and the magnetic dipole form factors are predicted, and the remaining form factors, the electric quadrupole and magnetic octupole, vanish, since they measure the deviation from the symmetric distribution. Nevertheless, it is still interesting to verify, as done in this work, whether a  $\Delta$  model calibrated by the  $N\Delta$  transition gives a good description of the lattice QCD extrapolation of the  $\Delta$  charge and magnetic dipole form factors to the physical region, and therefore determines uniquely the  $\Delta$  properties. We note that our results are true predictions of our model for the  $\Delta$  electromagnetic form factors, since the parameters of the constituent quark model were beforehand fixed by the nucleon and  $N\Delta$  electromagnetic transition form factors data only [1, 6]. No further adjusting of parameters or refitting to the  $\Delta$  observables was done.

Since we did not include D-waves in the  $\Delta$  wave function, the quality of the obtained description indicates that there is only a small percentage (less than 5%) of D-wave components in the  $\Delta$  wave function, as found in calculations of the electromagnetic observables for the  $N\Delta$  electromagnetic transition [7]. We note that the contribution of the D-states to the  $\Delta$  form factors comes predominantly from the transitions between the  $\Delta$  S-state and the  $\Delta$  D-states, and that we have checked that the correction from the D-states for the  $\Delta$  charge form factor is less than 7%, and for the magnetic dipole is less than 14%, for low  $Q^2$  ( $Q^2 < 1.0 \text{ GeV}^2$ ) [42]. With an even more realistic and smaller D-state admixture [43] these numbers decrease even further, to 2% and 11% respectively.

Another interesting point to explore is that, although the pion cloud is essential to describe the  $\gamma N \rightarrow \Delta$  transition reaction form factors, as shown in Ref. [6], there is not yet direct evidence about the importance of the pion cloud for the elastic channel ( $\gamma\Delta \rightarrow \Delta$ ). The validity of our approximation of neglecting the pion cloud is to be judged from the deviations of our results from the data.

The paper is organized in the following way: in Sec. II we introduce the general definitions for the  $\Delta$  form factors and in Sec. III we present the corresponding analytical expressions in the covariant spectator formalism used here. In Sec. IV we present numerical results and in Sec. V we draw conclusions.

## II. $\Delta$ ELECTROMAGNETIC FORM FACTORS

The electromagnetic interaction with a on-mass-shell  $\Delta$  isobar, for initial momentum  $P_-$  and final momentum  $P_+$ , can be parametrized in terms of the current [44, 45, 46]:

$$J^\mu = -\bar{w}_\alpha(P_+) \left\{ \left[ F_1^*(Q^2)g^{\alpha\beta} + F_3^*(Q^2)\frac{q^\alpha q^\beta}{4M_\Delta^2} \right] \gamma^\mu + \left[ F_2^*(Q^2)g^{\alpha\beta} + F_4^*(Q^2)\frac{q^\alpha q^\beta}{4M_\Delta^2} \right] \frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta} \right\} w_\beta(P_-). \quad (1)$$

where  $w_\alpha$  is the Rarita-Schwinger spin 3/2 state, associated with the spin projection  $s = \pm 1/2, \pm 3/2$ , which is not specified in the equation for the sake of simplicity. The functions  $F_i^*(Q^2)$  ( $i = 1, \dots, 4$ ) are the  $\Delta$  form factors. In particular  $F_1^*(0) = e_\Delta$ , where  $e_\Delta$  is the  $\Delta$  charge.

It is more convenient to use the multipole form factor functions, labeled in terms of the multipoles observed in the electromagnetic transitions. The  $\Delta$  form factors can be separated into the electric charge (E0) and quadrupole (E2) form factors, and magnetic dipole (M1) and octupole (M3) form factors, defined as [44, 45, 46, 47]:

$$G_{E0}(Q^2) = [F_1^* - \tau F_2^*] \left( 1 + \frac{2}{3}\tau \right) - \frac{1}{3} [F_3^* - \tau F_4^*] \tau (1 + \tau) \quad (2)$$

$$G_{M1}(Q^2) = [F_1^* + F_2^*] \left( 1 + \frac{4}{5}\tau \right) - \frac{2}{5} [F_3^* + F_4^*] \tau (1 + \tau) \quad (3)$$

$$G_{E2}(Q^2) = [F_1^* - \tau F_2^*] - \frac{1}{2} [F_3^* - \tau F_4^*] (1 + \tau) \quad (4)$$

$$G_{M3}(Q^2) = [F_1^* + F_2^*] - \frac{1}{2} [F_3^* + F_4^*] (1 + \tau) \quad (5)$$

where  $\tau = \frac{Q^2}{4M_\Delta^2}$ . The static magnetic dipole ( $\mu_\Delta$ ), electric quadrupole ( $Q_\Delta$ ) and magnetic octupole ( $O_\Delta$ ) are defined,

in the  $Q^2 = 0$  limit, as

$$\begin{aligned}\mu_\Delta &= \frac{e}{2M_\Delta} G_{M1}(0) \\ Q_\Delta &= \frac{e}{M_\Delta^2} G_{E2}(0) \\ O_\Delta &= \frac{e}{2M_\Delta^3} G_{M3}(0).\end{aligned}\tag{6}$$

### III. FORM FACTORS IN A S-WAVE MODEL

In this work we consider the wave function obtained within the covariant spectator theory, as proposed in Ref. [6]:

$$\Psi_\Delta(P, k) = -\psi_\Delta(P, k) (T \cdot \xi^{1*}) \varepsilon_P^\alpha w_\alpha(P).\tag{7}$$

In the previous equation  $\psi_\Delta$  is a scalar function that describes the momentum distribution of the quark-diquark system in terms of the  $\Delta$  and the diquark momenta,  $P$  and  $k$  respectively;  $\varepsilon_P^*$  is the polarization state associated with the diquark spin [2] (the polarization index was omitted);  $w_\beta$  is the Rarita-Schwinger state, as before, and  $T \cdot \xi^{1*}$  is the isospin operator that acts on a given Delta isospin state [isospin states are not explicitly included]. As discussed in Ref. [6], Eq. (7) describes a quark-diquark system with total angular momentum  $J = \frac{3}{2}$  and no orbital momentum (S-state only), with the diquark on-mass-shell, a defining condition of the covariant spectator theory. The scalar wave function  $\psi_\Delta(P, k)$  can be written as

$$\psi_\Delta(P, k) = \frac{N}{m_s(\alpha_1 + \chi)(\alpha_2 + \chi)^2},\tag{8}$$

where  $N$  is a normalization constant, and  $\chi$  is defined as

$$\chi = \frac{(M_\Delta - m_s)^2 - (P - k)^2}{M_\Delta m_s}.\tag{9}$$

The variables  $\alpha_1$  and  $\alpha_2$  represent momentum range parameters in  $1/(M_\Delta m_s)$  units. The particular dependence of  $\psi_\Delta$  in  $(P - k)^2$  given by Eq. (8) was taken for a good description of the  $N\Delta$  transition form factors (see Ref. [6]) in particular the magnetic dipole form factor, which dominates the reaction. The inclusion of the factor  $1/m_s$  in the scalar wave function (8) implies that the diquark mass scales out of the transition currents, leading to form factors independent of  $m_s$  [1]. From the fit to the data, we have obtained  $\alpha_1 = 0.290$  and  $\alpha_2 = 0.393$  (model II of Ref. [6]).

#### A. The current

In our model, the  $\gamma\Delta \rightarrow \Delta$  electromagnetic current in the relativistic impulse approximation becomes [2, 6]

$$J^\mu = 3 \sum_\lambda \int_k \bar{\Psi}_\Delta(P_+, k) j_I^\mu \Psi_\Delta(P_-, k),\tag{10}$$

where the quark current is defined as

$$j_I^\mu = \frac{1}{6}(f_{1+} + f_{1-\tau_3})\gamma^\mu + \frac{1}{2}(f_{2+} + f_{2-\tau_3})\frac{i\sigma^{\mu\nu}q_\nu}{2M},\tag{11}$$

and  $M$  is the nucleon mass.

The functions  $f_{1\pm}(Q^2)$  and  $f_{2\pm}(Q^2)$  describe respectively the Dirac and Pauli quark form factors that parametrize the structure of the constituent quarks. As in Refs. [1, 6] we used the form inspired on the vector meson dominance mechanism:

$$f_{1\pm}(Q^2) = \lambda + (1 - \lambda) \frac{m_v^2}{m_v^2 + Q^2} + c_\pm \frac{Q^2 M_h^2}{(M_h^2 + Q^2)^2}\tag{12}$$

$$f_{2\pm}(Q^2) = \kappa_\pm \left\{ d_\pm \frac{m_v^2}{m_v^2 + Q^2} + (1 - d_\pm) \frac{Q^2}{M_h^2 + Q^2} \right\}.\tag{13}$$

In the previous equations  $\lambda$  was adjusted to the charge number density in deep inelastic limit [1], which gave  $\lambda = 1.21$ . The variable  $m_v$  represents a vector meson ( $m_v = m_\rho \approx m_\omega$ ),  $M_h$  is a mass off an effective heavy vector meson that simulates the short range hadron structure, and  $\kappa_+$  ( $\kappa_-$ ) is the isoscalar (isovector) quark anomalous magnetic moment. The anomalous magnetic moments  $\kappa_\pm$  are related with the quark  $u$  and  $d$  anomalous magnetic moments through  $e_q \kappa_q = \frac{1}{6} \kappa_+ + \frac{\tau_3}{2} \kappa_-$ , where  $\tau_3$  is the isospin operator and  $e_q = \frac{1}{6} + \frac{\tau_3}{2}$  the quark charge, for  $q = u, d$  [1]. The functions  $f_{i\pm}(Q^2)$  ( $i = 1, 2$ ) are normalized according to  $f_{1\pm}(0) = 1$  and  $f_{2\pm}(0) = \kappa_\pm$ .

In this application we consider in particular the parametrization derived in Ref. [1]. The anomalous magnetic moments  $\kappa_\pm$ , were adjusted to reproduce the nucleon magnetic moments, corresponding to  $\kappa_+ = 1.639$  and  $\kappa_- = 1.823$ . The effective heavy meson mass is  $M_h = 2M$ , as in model II of Ref. [1]. The remaining parameters correspond also to the model II of this last work, constrained by a fit the the elastic nucleon form factor data. The vector meson dominance coefficients are respectively  $c_+ = 4.16$ ,  $c_- = 1.16$  and  $d_\pm = -0.686$ .

Using Eq. (7) the current in Eq. (10) reduces to

$$J^\mu = [\bar{w}_\alpha(P_+) A^\mu \Delta^{\alpha\beta} w_\beta(P_-)] I_\Delta, \quad (14)$$

where

$$A^\mu = 3 \sum_i T_i^\dagger j_I^\mu T_i, \quad (15)$$

$$I_\Delta = \int_k \psi_\Delta(P_+, k) \psi_\Delta(P_-, k), \quad (16)$$

and  $\Delta^{\alpha\beta}$  is the covariant sum in the diquark polarization for the case  $M_+ = M_- = M_\Delta$ ; see Refs. [1, 2, 6] for details. Explicitly one has

$$\Delta^{\alpha\beta} = -g^{\alpha\beta} - \frac{P_+^\alpha P_-^\beta}{M_\Delta^2} + 2 \frac{(P_+ + P_-)^\alpha (P_+ + P_-)^\beta}{4M_\Delta^2 + Q^2}. \quad (17)$$

The integral  $I_\Delta(Q^2)$  defined by Eq. (16) is normalized according to  $I_\Delta(0) = 1$ , such that the charge condition  $J^0 = e_\Delta$  holds ( $e_\Delta$  is the  $\Delta$  charge) for  $Q^2 = 0$ .

Performing the sum in the isospin operators, and after the spin algebra by using the properties of the  $w_\alpha(P)$  states, we can furthermore write  $J^\mu$  as

$$\begin{aligned} J^\mu = & -\bar{w}_\alpha(P_+) \left\{ \left[ \tilde{e}_\Delta \left( g^{\alpha\beta} + 2 \frac{q^\alpha q^\beta}{4M_\Delta^2 + Q^2} \right) I_\Delta \right] \gamma^\mu \right\} w_\beta(P_-) \\ & -\bar{w}_\alpha(P_+) \left\{ \left[ \tilde{\kappa}_\Delta \left( g^{\alpha\beta} + 2 \frac{q^\alpha q^\beta}{4M_\Delta^2 + Q^2} \right) I_\Delta \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M} \right\} w_\beta(P_-). \end{aligned} \quad (18)$$

The isospin dependent functions  $\tilde{e}_\Delta$  and  $\tilde{\kappa}_\Delta$  are respectively

$$\tilde{e}_\Delta(Q^2) = \frac{1}{2} [f_{1+}(Q^2) + f_{1-}(Q^2) \bar{T}_3] \quad (19)$$

$$\tilde{\kappa}_\Delta(Q^2) = \frac{1}{2} [f_{2+}(Q^2) + f_{2-}(Q^2) \bar{T}_3], \quad (20)$$

and  $\bar{T}_3$  is the isospin- $\frac{3}{2}$  matrix defined as

$$\bar{T}_3 = 3 \sum_i T_i^\dagger \tau_3 T_i = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}. \quad (21)$$

The  $T_i$  ( $i = x, y, z$ ) matrices represent the isospin- $\frac{3}{2}$  to isospin- $\frac{1}{2}$  transition operators [6, 13, 45]. Note that in the limit  $Q^2 = 0$ ,  $\tilde{e}_\Delta$  is just the  $\Delta$  charge

$$e_\Delta = \frac{1}{2} (1 + \bar{T}_3). \quad (22)$$

For the numerical evaluation of the form factors we took one of the models presented in Refs. [1, 6] which is not isospin symmetric. Isospin symmetry is broken through  $f_{1+} \neq f_{1-}$  and  $f_{2+} \neq f_{2-}$ . Isospin symmetry would imply  $\kappa_+ = \kappa_- = \kappa$ . In this case one has  $\tilde{\kappa}_\Delta = e_\Delta \kappa$  for  $Q^2 = 0$ . For the simplest models applied in Ref. [1] to the nucleon, the isospin symmetry is exact for the Dirac quark current ( $f_{1+} \equiv f_{1-}$ ), giving an almost zero electrical form factor for the neutron. If in Ref. [1] we would have taken  $f_{2+} \equiv f_{2-}$ , exact isospin symmetry would be observed, and  $G_{En}$  would vanish as well. This is why in the present application we used models which are not isospin symmetric.

$G_{M1}(0)$	$\Delta^{++}$	$\Delta^+$	$\Delta^0$	$\Delta^-$
Exp.	$7.34 \pm 2.49$	$3.54^{+4.59}_{-4.72}$	-	-
SU(6)	7.31	3.65	0	-3.65
Rel QM [25]	6.24	3.12	0	-3.12
QCD SR [28]	$5.76 \pm 1.05$	$2.88 \pm 0.52$	0	$-2.88 \pm 0.52$
QCD SR [29]	$5.41 \pm 1.70$	$2.71 \pm 0.85$	0	$-2.71 \pm 0.85$
ChQSM [30]	7.06	3.48	-0.10	-3.69
HBChPT [31]	$6.24 \pm 0.52$	$2.75 \pm 0.26$	$-0.22 \pm 0.05$	$-2.95 \pm 0.33$
Lattice [36]	$6.43 \pm 0.80$	$3.22 \pm 0.41$	0	$-3.22 \pm 0.41$
Lattice [37]	$6.54 \pm 0.73$	$3.26 \pm 0.35$	0.079	$-3.22 \pm 0.35$
Lattice [38]	$6.86 \pm 0.24$	$1.27 \pm 0.10$	$-0.046 \pm 0.003$	$-3.90 \pm 0.25$
Lattice [39]		$3.04 \pm 0.21$		
Spectator	6.71	3.29	-0.12	-3.54

TABLE I: Summary of existing experimental and theoretical results for  $G_{M1}(0)$ . The conversion between  $\mu_\Delta$  in  $\mu_N$  unities to  $G_{M1}(0)$  is  $\frac{M_\Delta}{M}\mu_\Delta$ . The Table shows results based on SU(6) static quark structure [22], on a relativistic quark model (Rel QM) [25], QCD sum rules (QCD SR) [28, 29], Chiral Quark-Soliton Models (ChQSM) [30], Heavy Baryon Chiral Perturbation Theory (HBChPT) [31] and extrapolations from Lattice QCD [36, 37, 38, 39]. Our results are labeled "Spectator".

## B. Spectator Form Factors

Comparing the currents (1) and (18) we conclude that

$$F_1^* = \tilde{e}_\Delta I_\Delta \quad (23)$$

$$F_3^* = \eta \tilde{e}_\Delta I_\Delta = \eta F_1^* \quad (24)$$

$$F_2^* = \frac{M_\Delta}{M} \tilde{\kappa}_\Delta I_\Delta \quad (25)$$

$$F_4^* = \frac{M_\Delta}{M} \eta \tilde{\kappa}_\Delta I_\Delta = \eta F_2^*, \quad (26)$$

where  $\eta = \frac{2}{1+\tau}$ .

Considering the relations (23)-(26), we can write  $F_3^* - \tau F_4^* = \eta(F_1^* - \tau F_2^*)$  and  $F_3^* + F_4^* = \eta(F_1^* + F_2^*)$ . Then, by using the definitions (2)-(3), we obtain

$$\begin{aligned} G_{E0}(Q^2) &= \left( \tilde{e}_\Delta - \tau \frac{M_\Delta}{M} \tilde{\kappa}_\Delta \right) I_\Delta \\ G_{M1}(Q^2) &= \left( \tilde{e}_\Delta + \frac{M_\Delta}{M} \tilde{\kappa}_\Delta \right) I_\Delta. \end{aligned} \quad (27)$$

For the electric quadrupole  $G_{E2}$  and magnetic octupole  $G_{M3}$ , one has  $G_{E2} \equiv 0$  and  $G_{M3} \equiv 0$ , and the electric quadrupole  $Q_\Delta$  and magnetic octupole  $O_\Delta$  moments vanish. This particular result follows directly from our restriction to  $L = 0$  of the orbital angular momentum of the quark-diquark in the  $\Delta$  wave function. A better description would require other states, in particular D-state components of the wave function. Still, from the  $N\Delta$  data, one concludes that the percentage of these components is small [7].

## IV. NUMERICAL RESULTS

The model dependence of the  $\Delta$  form factors comes from the quark form factors  $f_{1\pm}, f_{2\pm}$  given by Eqs. (12)-(13), and from the particular form for the scalar wave function  $\psi_\Delta$  given by Eq. (8), as introduced in Refs. [1, 6]. The quark current is parametrized by three coefficients ( $c_+, c_-$  and  $d_+ = d_-$ ), and the scalar wave function by two range parameters ( $\alpha_1$  and  $\alpha_2$ ). Apart from these five parameters fixed in Refs. [1, 6], there are no adjusted parameters involved in the current calculation. Then, our results for the  $\Delta$  form factors E0 and M1, given by Eqs. (27), and the magnetic moment  $\mu_\Delta$ , in particular, are true predictions.

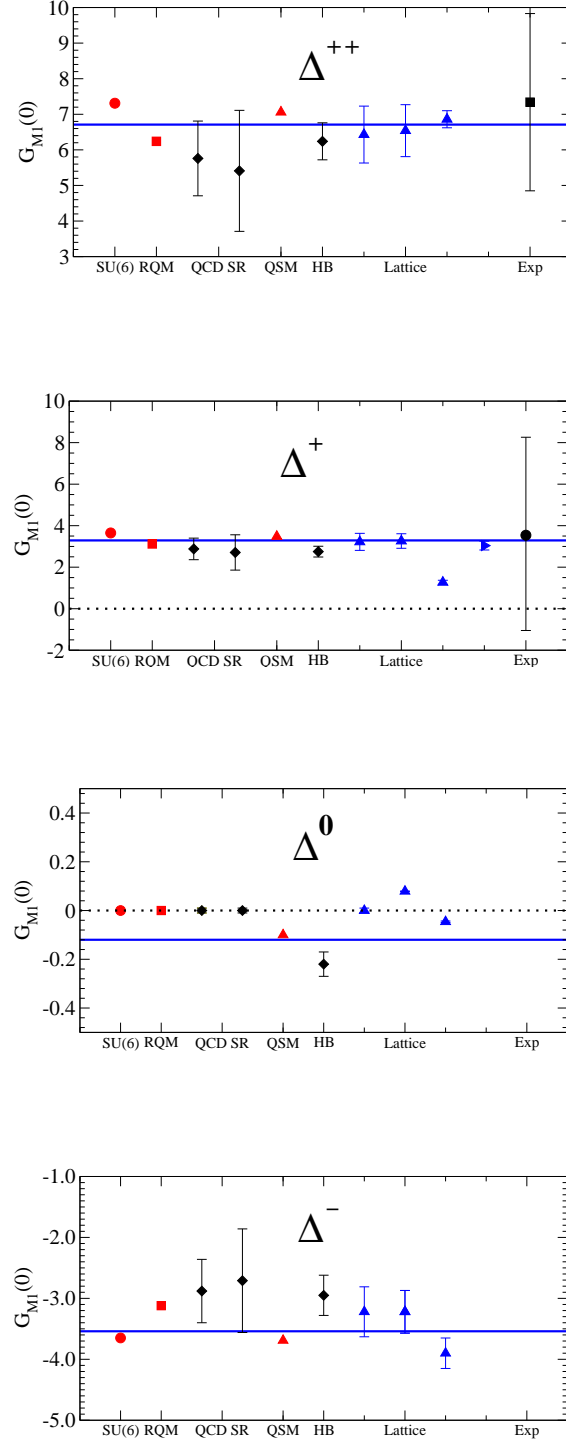


FIG. 1:  $G_{M1}(0)$  in different descriptions. RQM holds for Relativistic Quark Model, QSM for ChQSM (Chiral Quark-Soliton Model) and HB for HBChPT (Heavy Baryon Chiral Perturbation Theory). Lattice results corresponds respectively to Refs. [36, 37, 38, 39].

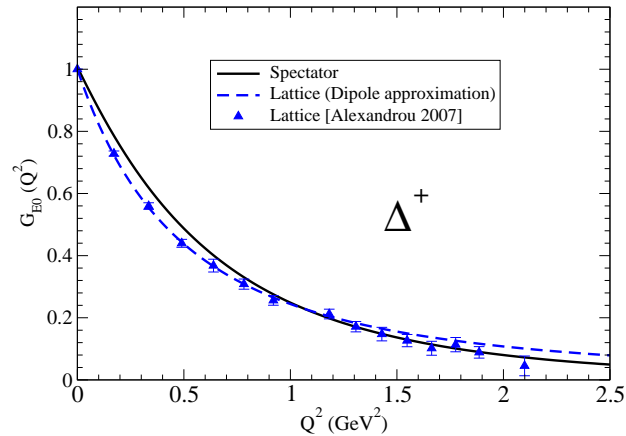


FIG. 2: Electric charge form factors for the  $\Delta^+$ . Lattice data from Refs. [39, 49]. The line described as Lattice (dipole approximation) is a result of a dipole fit from the same references and corresponds to  $r_{E0} = 0.691$  fm.

We evaluate the  $\Delta$  electromagnetic form factors numerically by using Eqs. (27) for both  $Q^2 = 0$  and  $Q^2 \neq 0$ . The first case gives us the static properties of the  $\Delta$ . The second case gives us information about the dynamical properties of the  $\Delta$ , unfolding the dependence of the form factors on  $Q^2$ .

### A. Static properties

The static properties of the  $\Delta$ , like the charge and magnetic moment, are completely fixed by the static properties of the quarks (charge and magnetic moments  $\kappa_{\pm}$ ), constrained by the nucleon charge and magnetic moment.

As mentioned above, for a spherically symmetric  $\Delta$  wave function (S-wave states) only the electrical charge  $G_{E0}$  and magnetic dipole  $G_{M1}$  form factors are different from zero for  $Q^2 = 0$ . Since the charge of the  $\Delta$  is well known, the relevant information at  $Q^2 = 0$  comes from the form factor  $G_{M1}$  alone. Therefore we will focus on this observable.

A list of results for  $G_{M1}(0)$  obtained from different frameworks is presented in Table I. Although additional results can be found in Refs. [25, 26, 27, 28, 29, 32, 33, 34], the cases shown in Table I represent well the different approaches found in the literature. The experimental result was taken from the Particle Data Group [18]. The label "Spectator" denotes model II of Refs. [1, 6] used here, and therefore our results. We present the results for  $G_{M1}(0)$  instead of the magnetic moment  $\mu_{\Delta}$  in order to simplify the direct comparison with the results for the form factor at  $Q^2 \neq 0$  in the next section.

All lattice QCD results shown were obtained in the quenched approximation. In quenched calculations no disconnected diagrams are considered [36], and important features of quark-antiquark pair creation and pion loop effects are suppressed [48]. Lattice simulations for heavy pion masses are performed in Refs. [36, 38, 39]. In Refs. [36, 39] the form factors are determined using a linear extrapolation of  $m_{\pi}^2$  ( $m_{\pi}$  is the pion mass in lattice) to the physical region. In Ref. [38] alternative empirical extrapolations are tested. Reference [37] presents a re-analysis of the lattice data of [36] using an effective  $\chi$ PT. A similar analysis was also presented in Ref. [13]. The  $\chi$ PT effects estimated in Refs. [13, 37] show that the linear extrapolation, used in particular in reference [39] for the case  $Q^2 \neq 0$ , is not totally adequate. For this reason we favor the comparison with lattice extrapolation based on  $\chi$ PT [37]. The weak dependence of the data on the pion mass suggests nevertheless that a linear extrapolation can be taken as good first estimate, meaning that the non-analytical terms bring only small effects, in contrast with what is required for the nucleon elastic form factors and  $N\Delta$  transition form factors. Unquenched lattice simulations presented in Ref. [40], and planned extrapolations based on  $\chi$ PT can help to clarify the situation [39, 40].

For a clear comparison of our results with the different calculations we plot in Fig. 1 all results, including their uncertainty bars. Fig. 1 depicts therefore in a graphic form the results of Table I. From the figure we conclude that, excluding for now the  $\Delta^0$  state, all results are consistent with most of the predictions within one  $\sigma$  deviation. In particular, for the magnetic moment of the  $\Delta^{++}$ , we get a very reasonable value. Our results are also consistent with

the central value of the magnetic moment of the  $\Delta^+$ , although negative values cannot be excluded due to the error bars.

There is a remarkable exception to the success obtained in comparing our results to experimental or lattice data for the  $\Delta$  baryon: the lattice estimation of Ref. [38] for the  $\Delta^+$ . However, according to Ref. [48] the lower prediction of Lee *et. al.* [38] is an artifact of the quenched calculation that, for lower values of the pion mass, pushes the magnetic moment of the  $\Delta^+$  away from the proton magnetic moment.

As for the  $\Delta^0$  state, it is more difficult to extract definitive conclusions. From Table I, the sign of the  $\Delta^0$  magnetic moment is strongly model dependent, and its magnitude small when compared to the other  $\Delta$  isospin states. Note that there is no direct experimental data, since the short life time and neutral character of the  $\Delta^0$  make difficult the determination of its magnetic moment. Probably the most significant experimental information comes indirectly from measurements on the  $\gamma n \rightarrow \Delta^0$  reaction.

## B. Dynamical properties

For low  $Q^2$  we can approximate any form factor  $G_a$  by a dipole form

$$G_a(Q^2) = G_a(0) \left( \frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^2 \quad (28)$$

where  $a = E0, M1, E2, M3$ , and  $\Lambda$  is an adequate momentum range (cutoff). In these conditions the low  $Q^2$  behavior is given by the average squared radius  $r_a^2$ :

$$r_a^2 = - \frac{6}{G_a(0)} \frac{dG_a}{dQ^2} \Big|_{Q^2=0}. \quad (29)$$

The momentum dependence of the form factors was for the first time considered in a systematic way in Refs. [39, 40]. In Ref. [39] the  $\Delta^+$  state was considered explicitly, and the lattice data for the charge form factor  $G_{E0}(Q^2)$  well simulated by a dipole form factor with  $r_{E0} = 0.691 \pm 0.006$  fm. This result is also consistent with the lattice results of Ref. [37],  $0.63 \pm 0.07$  fm. In Fig. 2 we compare our prediction with the lattice data from Refs. [39, 49] and also with the dipole form extracted from the data [39]. Our prediction does not follow exactly a simple dipole form in the range  $[0, 2.5]$  GeV<sup>2</sup>, but nonetheless does not deviate much from the lattice data. Still, we predict a slower falloff corresponding to  $r_{E0} = 0.57$  fm, instead of  $r_{E0} = 0.69$  fm. Therefore, relatively to the lattice calculations, we predict an higher concentration of charge at the origin. Note however, that both lattice calculations and ours underestimate the proton electric radius, leading to  $r_{Ep} = 0.89$  fm [1], which implies that the  $\Delta^+$  has a larger charge density near the origin than the proton. This result contradicts the estimations based on simple quark models governed by the hyperfine interaction, where for the  $\Delta$  there is a repulsive quark-quark interaction in the spin-triplet state which prevents charge concentration at the origin, differently than for the proton case. We would then expect a larger extension of the charge in the  $\Delta$  [50]. Interestingly, this expectation is contradicted by the recent quenched lattice QCD simulations [41], where the  $\Delta^+$  charge radius is always smaller than the proton charge radius. To clarify this point we need still to wait for unquenched lattice QCD simulations for pion masses below the inelastic cut ( $m_\pi < M_\Delta - M$ ). At any rate, our result is consistent with the speculation that, contrarily to the  $N\Delta$  and  $\gamma N \rightarrow N$  transitions (see models III and IV of Ref. [1]), the pion cloud effect is not as important for the direct reaction  $\gamma\Delta \rightarrow \Delta$  form factors,  $G_{E0}$  and  $G_{M1}$ , since that reaction may be less affected by the behavior of the  $\Delta$  as a  $\pi$ -nucleon system.

Next we discuss the dipole magnetic form factor. In Ref. [39] only the state  $\Delta^+$  was considered, and it was suggested that isospin symmetry can be used to obtain the results for the other charge states. Applying this principle, we consider

$$G_{M1}(\Delta^{++}) = 2G_{M1}(\Delta^+) \quad (30)$$

$$G_{M1}(\Delta^-) = -G_{M1}(\Delta^+), \quad (31)$$

in order to generalize the lattice data of Ref. [39]. Although our model breaks the isospin symmetry explicitly ( $f_{1+} \neq f_{1-}$  and  $f_{2+} \neq f_{2-}$ ), the extent of this violation is very small. Therefore the comparison of this representation with our results is still reasonable (the exact isospin symmetry gives  $G_{M1}(\Delta^0) \equiv 0$ ).

The results of  $G_{M1}$  for each  $\Delta$  state are presented in Fig. 3. We compare our results with the chiral extrapolation of the lattice data of Cloet *et. al.* [37], with the  $\Delta^+$  lattice data from Refs. [39, 49], and with the dipole approximation for the lattice results, assuming exact isospin symmetry as in Eqs. (30)-(31), using  $r_{M1} = (0.642 \pm 0.038)$  fm from those works. For the  $\Delta^+$ , we estimate  $r_{M1} = 0.61$  fm in agreement with [39, 49]. Our results are consistent with



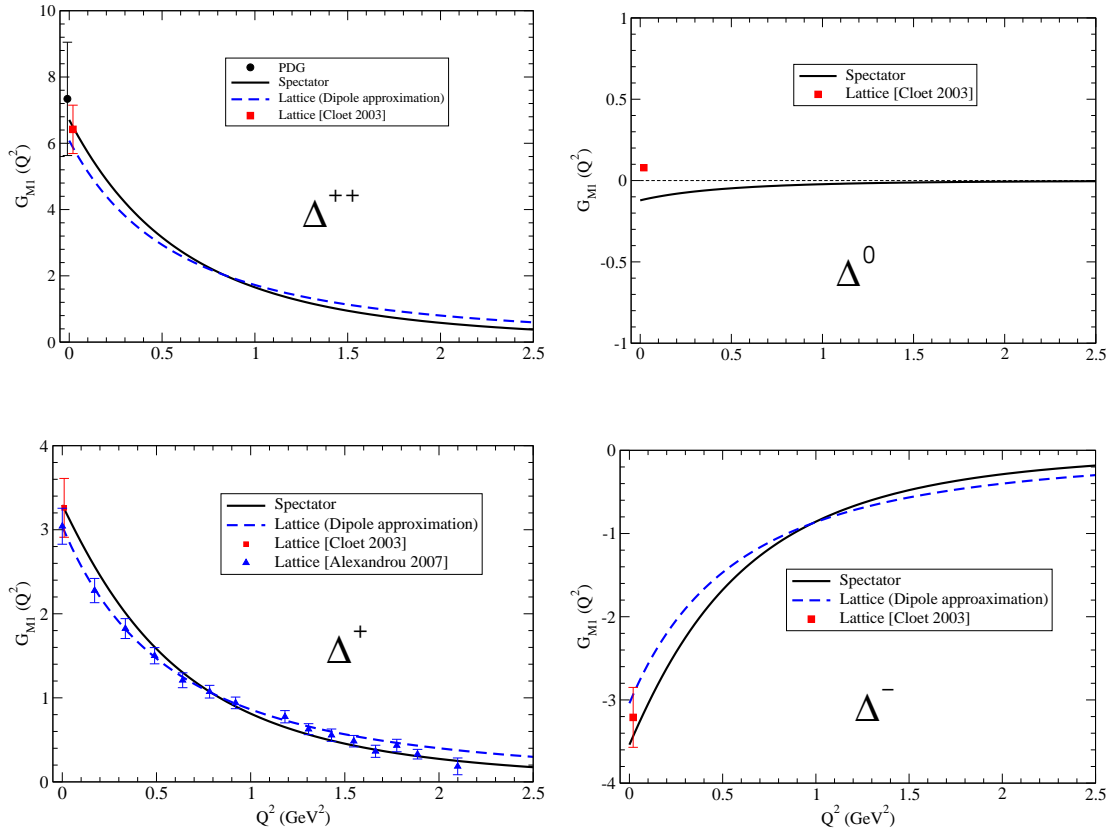


FIG. 3: Magnetic dipole form factor for  $\Delta^{++}$ ,  $\Delta^+$ ,  $\Delta^0$  and  $\Delta^-$ . Lattice data from [37] for  $Q^2 = 0$  and Refs. [39, 49] for  $\Delta^+$ . The dipole line presented for  $\Delta^{++}$  and  $\Delta^-$  is derived from the lattice data of Refs. [39, 49], assuming the isospin symmetry and  $r_{M1} = (0.642 \pm 0.038)$  fm. Experimental data for  $\Delta^{++}$  from Ref. [18]. The experimental value for  $\Delta^+$  is not presented (see Fig. 1).

the lattice data from Alexandrou *et al.* [39, 49] considering the error bands. As for  $\Delta^-$ , clearly our results are in agreement with those of Ref. [37] (for  $Q^2 = 0$ ) and are very similar to the dipole approximation. As for  $\Delta^0$  no dipole approximation line is shown, because exact isospin symmetry predicts exactly  $G_{M1} \equiv 0$ , and in fact both our result for  $G_{M1}(Q^2)$  and the lattice data from [37] are very small when compared with the corresponding results for the charged  $\Delta$ . We note that is a difference of sign relatively to Ref. [37]. However, this difference of sign is not a problem here, since in that reference  $G_{M1}(0)$  was extrapolated from  $G_{M1} \equiv 0$  for heavy pion masses, and no error estimate was given (see Table I).

## V. CONCLUSIONS

There is almost no direct experimental information about the electromagnetic structure of the  $\Delta$  system. Only for the  $\Delta^{++}$  there is some conclusive, although still very limited data. For this reason information on the  $\Delta$  structure for  $Q^2 \neq 0$  has to rely on theoretical models fully constrained by lattice QCD data, or, inversely, on lattice data extrapolations conveniently probed in the physical limit. Several models can be consistent with the data but different experiments are planned to improve the accuracy of the data for the  $\Delta^{++}$  and  $\Delta^+$ , which will enable a finer model selection. A quark model successful in the description of the nucleon elastic form factor data and of the dominant contribution to the  $N\Delta$  electromagnetic transition was taken here to describe the  $\Delta$  form factors, without extra parameter fitting [1, 6]. Our results were compared directly to the (still) scarce existing data as well as to the lattice extrapolations to the physical region.

In particular, our results compare well with the first quenched lattice study of the  $\Delta$  form factors for finite  $Q^2$  [39]. In this work we consider only the valence quark degrees of freedom. Although pion cloud effects are fundamental for inelastic reactions, namely the  $N\Delta$  transition [6], our results were obtained without the explicit consideration of pion

cloud, turning out to be consistent with the experimental data for the  $\Delta$  magnetic dipole moment, as well as with the extrapolation from lattice QCD [39] results. As for the charge distribution we predict a higher concentration of charge near the  $\Delta$  center of mass.

There are two aspects to be improved in the future for an even more significant comparison. On one side, lattice simulations are, until very recently, restricted to quenched approximations (Ref. [40] is the first unquenched calculation). Also they depend on extrapolations to the physical region that are mostly linear, without control of the non-analytical terms. On the other side, our model takes at this stage a simple spherically symmetric  $\Delta$  wave function. For this reason the subleading form factors vanish. The extension of the model with the inclusion of the D-states is being investigated meanwhile [42]. Upon improvement of both these two aspects in each of the sectors compared here, the tool developed and tested in this work will allow one to investigate in finer detail the failure of the lattice calculations in describing the form factors for the  $N\Delta$  electromagnetic transition.

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- [1] F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. C **77**, 015202 (2008) [arXiv:nucl-th/0606029].
  - [2] F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. C **77**, 035203 (2008).
  - [3] M. K. Jones *et al.* [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. **84**, 1398 (2000); O. Gayou *et al.* [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. **88**, 092301 (2002); V. Punjabi *et al.*, Phys. Rev. C **71**, 055202 (2005) [Erratum-ibid. C **71**, 069902 (2005)] [arXiv:nucl-ex/0501018].
  - [4] M. Gockeler, T. R. Hemmert, R. Horsley, D. Pleiter, P. E. L. Rakow, A. Schafer and G. Schierholz [QCDSF Collaboration], Phys. Rev. D **71**, 034508 (2005) [arXiv:hep-lat/0303019].
  - [5] P. Wang, D. B. Leinweber, A. W. Thomas and R. D. Young, Phys. Rev. D **75**, 073012 (2007) [arXiv:hep-ph/0701082].
  - [6] G. Ramalho, M. T. Peña and F. Gross, Eur. Phys. J. A **36**, 329 (2008) [arXiv:hep-ph/0803.3034].
  - [7] G. Ramalho, M. T. Peña and F. Gross, Phys. Rev. D **78**, 114017 (2008) [arXiv:0810.4126 [hep-ph]].
  - [8] T. Sato and T. S. H. Lee, Phys. Rev. C **54**, 2660 (1996) [arXiv:nucl-th/9606009]; T. Sato and T. S. H. Lee, Phys. Rev. C **63**, 055201 (2001) [arXiv:nucl-th/0010025].
  - [9] S. S. Kamalov, S. N. Yang, D. Drechsel, O. Hanstein and L. Tiator, Phys. Rev. C **64**, 032201 (2001) [arXiv:nucl-th/0006068]; S. S. Kamalov and S. N. Yang, Phys. Rev. Lett. **83**, 4494 (1999) [arXiv:nucl-th/9904072].
  - [10] U. G. Meissner, AIP Conf. Proc. **904**, 142 (2007) [arXiv:nucl-th/0701094].
  - [11] C. Alexandrou, Ph. de Forcrand, H. Neff, J. W. Negele, W. Schroers and A. Tsapalis, Phys. Rev. Lett. **94**, 021601 (2005) [arXiv:hep-lat/0409122].
  - [12] C. Alexandrou, G. Koutsou, H. Neff, J. W. Negele, W. Schroers and A. Tsapalis, Phys. Rev. D **77**, 085012 (2008) [arXiv:0710.4621 [hep-lat]].
  - [13] V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. Lett. **95**, 232001 (2005) [arXiv:hep-ph/0508060]; V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. D **73**, 034003 (2006) [arXiv:hep-ph/0512244].
  - [14] T. A. Gail and T. R. Hemmert, Eur. Phys. J. A **28**, 91 (2006) [arXiv:nucl-th/0512082].
  - [15] A. Bosshard *et al.*, Phys. Rev. D **44** 1962 (1991); C. A. Meyer *et al.*, Phys. Rev. D **38** 754 (1988).
  - [16] B. M. K. Nefkens *et al.*, Phys. Rev. D **18**, 3911 (1978).
  - [17] L. Heller, S. Kumano, J. C. Martinez and E. J. Moniz, Phys. Rev. C **35**, 718 (1987); D. H. Lin, M. K. Liou and Z. M. Ding, Phys. Rev. C **44**, 1819 (1991); P. Pascual and R. Tarrach, Nucl. Phys. B **134**, 133 (1978);
  - [18] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).
  - [19] G. Lopez Castro and A. Mariano, Nucl. Phys. A **697**, 440 (2002) [arXiv:nucl-th/0010045].
  - [20] M. Kotulla *et al.*, Phys. Rev. Lett. **89**, 272001 (2002) [arXiv:nucl-ex/0210040].
  - [21] V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. Lett. **94**, 102003 (2005) [arXiv:nucl-th/0412113].
  - [22] W. T. Chiang, M. Vanderhaeghen, S. N. Yang and D. Drechsel, Phys. Rev. C **71**, 015204 (2005) [arXiv:hep-ph/0409078]; D. Drechsel and M. Vanderhaeghen, Phys. Rev. C **64**, 065202 (2001) [arXiv:hep-ph/0105060].
  - [23] M. Kotulla, AIP Conf. Proc. **904**, 203 (2007).
  - [24] A. J. Buchmann, E. Hernandez and A. Faessler, Phys. Rev. C **55**, 448 (1997) [arXiv:nucl-th/9610040]; A. J. Buchmann, J. A. Hester and R. F. Lebed, Phys. Rev. D **66**, 056002 (2002) [arXiv:hep-ph/0205108]; A. J. Buchmann and E. M. Henley, Phys. Rev. D **65**, 073017 (2002); A. J. Buchmann and R. F. Lebed, Phys. Rev. D **67**, 016002 (2003) [arXiv:hep-ph/0207358].
  - [25] F. Schlumpf, Phys. Rev. D **48**, 4478 (1993) [arXiv:hep-ph/9305293].
  - [26] G. Wagner, A. J. Buchmann and A. Faessler, J. Phys. G **26**, 267 (2000).
  - [27] C. S. An, Q. B. Li, D. O. Riska and B. S. Zou, Phys. Rev. C **74**, 055205 (2006) [Erratum-ibid. C **75**, 069901 (2007)] [arXiv:nucl-th/0610009].

- [28] F. X. Lee, Phys. Rev. D **57**, 1801 (1998) [arXiv:hep-ph/9708323]; F. X. Lee, Phys. Lett. B **419**, 14 (1998) [arXiv:hep-ph/9707411].
- [29] T. M. Aliev, A. Ozpineci and M. Savci, Nucl. Phys. A **678**, 443 (2000) [arXiv:hep-ph/0002228].
- [30] H. C. Kim and M. Praszalowicz, Phys. Lett. B **585**, 99 (2004) [arXiv:hep-ph/0308242]; H. C. Kim, M. Praszalowicz and K. Goetze, Phys. Rev. D **57**, 2859 (1998) [arXiv:hep-ph/9706531].
- [31] M. N. Butler, M. J. Savage and R. P. Springer, Phys. Rev. D **49**, 3459 (1994) [arXiv:hep-ph/9308317].
- [32] C. Lorce, "Exotic and Non-Exotic Baryon Properties on Light Cone", PhD Thesis:  
[http://www.theo.phys.ulg.ac.be/PTF/THESES\\_files/lorce/PhDthesis.html](http://www.theo.phys.ulg.ac.be/PTF/THESES_files/lorce/PhDthesis.html) (2008).
- [33] K. Hashimoto, T. Sakai and S. Sugimoto, Prog. Theor. Phys. **120**, 1093 (2008) [arXiv:0806.3122 [hep-th]].
- [34] A. I. Machavariani and A. Faessler, arXiv:0804.1322 [nucl-th]; A. I. Machavariani, A. Faessler and A. J. Buchmann, Nucl. Phys. A **646**, 231 (1999) [Erratum-ibid. A **686**, 601 (2001)].
- [35] A. I. Machavariani and A. Faessler, arXiv:0809.1303 [nucl-th].
- [36] D. B. Leinweber, T. Draper and R. M. Woloshyn, Phys. Rev. D **46**, 3067 (1992) [arXiv:hep-lat/9208025].
- [37] I. C. Cloet, D. B. Leinweber and A. W. Thomas, Phys. Lett. B **563**, 157 (2003) [arXiv:hep-lat/0302008]; I. C. Cloet, D. B. Leinweber and A. W. Thomas, arXiv:nucl-th/0211027.
- [38] F. X. Lee, R. Kelly, L. Zhou and W. Wilcox, Phys. Lett. B **627**, 71 (2005) [arXiv:hep-lat/0509067].
- [39] C. Alexandrou, T. Korzec, T. Leontiou, J. W. Negele and A. Tsapalis, PoS **LAT2007**, 149 (2007) [arXiv:0710.2744 [hep-lat]].
- [40] C. Alexandrou *et al.*, Phys. Rev. D **79**, 014507 (2009) [arXiv:0810.3976 [hep-lat]].
- [41] S. Boinepalli, D. B. Leinweber, P. J. Moran, A. G. Williams, J. M. Zanotti and J. B. Zhang, arXiv:0902.4046 [hep-lat].
- [42] G. Ramalho, Franz Gross and M. T. Peña, work in preparation.
- [43] G. Ramalho and M. T. Peña, arXiv:0901.4310 [hep-ph].
- [44] S. Nozawa and D. B. Leinweber, Phys. Rev. D **42**, 3567 (1990).
- [45] V. Pascalutsa, M. Vanderhaeghen and S. N. Yang, Phys. Rept. **437**, 125 (2007) [arXiv:hep-ph/0609004].
- [46] M. Vanderhaeghen, private communication.
- [47] H. J. Weber and H. Arenhovel, Phys. Rept. **36** (1978) 277.
- [48] R. D. Young, D. B. Leinweber and A. W. Thomas, Nucl. Phys. Proc. Suppl. **128**, 227 (2004) [arXiv:hep-lat/0311038]; D. B. Leinweber, A. W. Thomas, A. G. Williams, R. D. Young, J. M. Zanotti and J. B. Zhang, Nucl. Phys. A **737**, 177 (2004) [arXiv:nucl-th/0308083].
- [49] C. Alexandrou, private communication.
- [50] M. M. Giannini, Rept. Prog. Phys. **54**, 453 (1990).